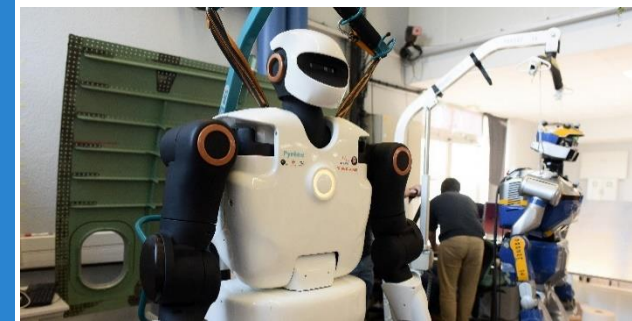


European  
Commission

Horizon 2020  
European Union funding  
for Research & Innovation

# Pinocchio

## Fast forward & inverse dynamics

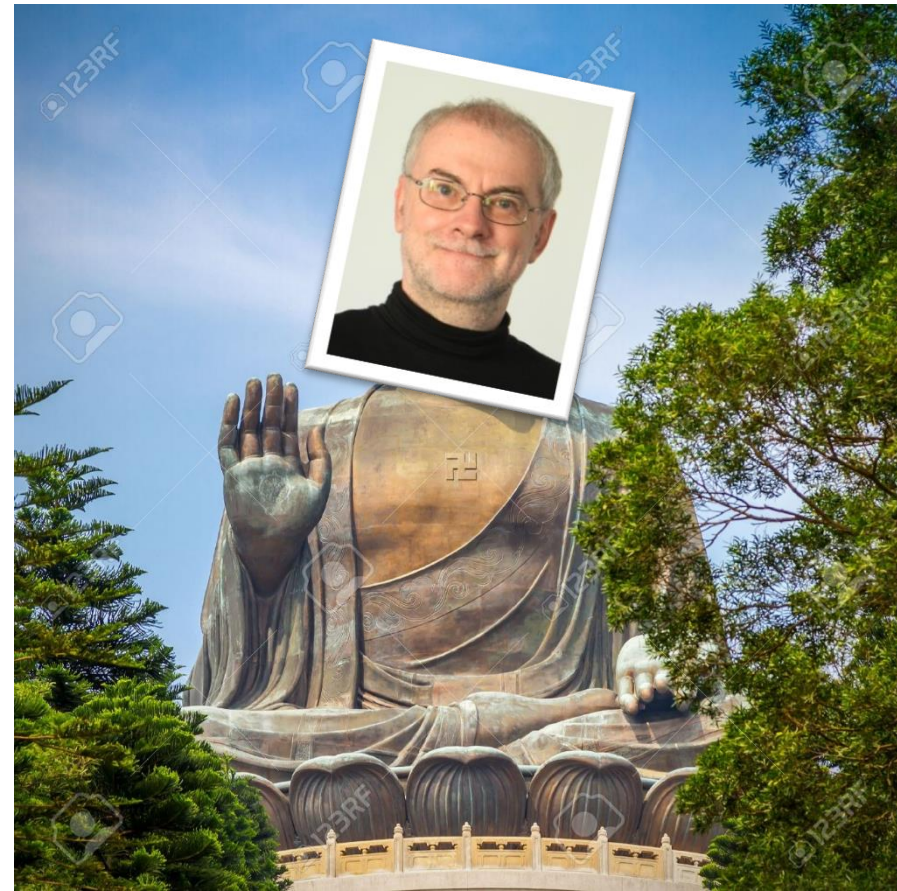


Nicolas Mansard  
(CNRS)

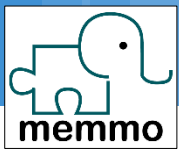




Justin Carpentier (INRIA)

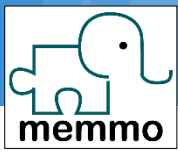


Roy Featherstone (IIT)



- ❑ Web site
  - ❑ <https://stack-of-tasks.github.io/pinocchio>
- ❑ Doxygen
  - ❑ Documentation tab on github.io
- ❑ Tutorials:
  - ❑ Practical exercises in the documentation
  
- ❑ Also use the ? In Python

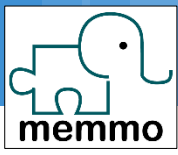




# Contributing to Pinocchio

- ❑ GitHub project
  - ❑ <https://github.com/stack-of-tasks/pinocchio>
  
- ❑ Post issues for contributing
  
- ❑ We are looking for doc-devs!
  - ❑ Feedback some material as a thank-you note
  - ❑ In the doc: “examples” is waiting for you





## □ C++ Library

- Fast, careful implementation
- Using curiously recursive template pattern (CRTP)
- You likely don't want to develop code there
- Using it is not so complex (think Eigen)

## □ Python bindings

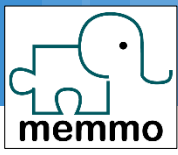
- A 1-to-0.99 map from C++ API to Python API
- Start by developing in Python
- Beware of the lack of accuracy ... speed is ok





- ❑ Pinocchio is a modeling library
  - ❑ Not an application
  - ❑ Not a solver
  - ❑ Some key features directly available
  
- ❑ You don't want the solver inside Pinocchio
  - ❑ Inverse dynamics: TSID
  - ❑ Planning and contact planning: HPP
  - ❑ Optimal control: Crocodyl
  - ❑ Optimal estimation, reinforcement learning, inverse kinematics, contact simulation ...





# List of features

- ❑ URDF parser
- ❑ Forward kinematics and Jacobians
- ❑ Mass, center of mass and gen.inertia matrix
- ❑ Forward and inverse dynamics
- ❑ Model display (with Gepetto-viewer)
- ❑ Collision detection and distances (with HPP-FCL)
- ❑ Derivatives of kinematics and dynamics
- ❑ Type templatization and code generation

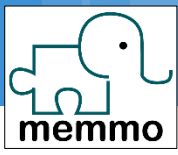


- ❑ Pinocchio for
  - ❑ Computing the inertia matrix, jacobians, kinematics
- ❑ Formulation of tasks
- ❑ Contact models
- ❑ QP resolution



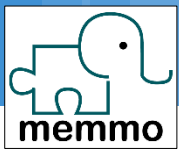
- ❑ Pinocchio for
  - ❑ Kinematics and dynamics
  - ❑ And their derivatives
  - ❑ Display with Gepetto-viewer
  
- ❑ DDP optimizer
- ❑ Task/cost formulation



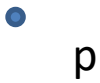


- ❑ Pinocchio for
  - ❑ Geometry, collision (hpp-fcl)
  - ❑ Projectors with inverse kinematics
  - ❑ Balance constraint with dynamics
- ❑ Pinocchio encapsulated in hpp-Pinocchio
- ❑ Stochastic exploration algorithm (RRT)
- ❑ Contact checking
- ❑ Re-arrangement algorithms

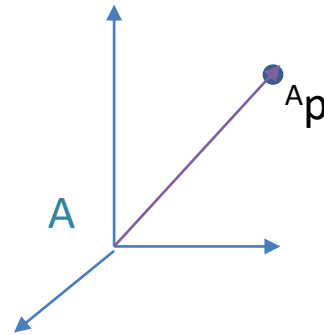




# Representing the physical world



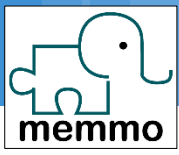
This is a point



This is not a point

This is the representation of a point





- ❑ Pinocchio is a model
  - ❑ Of course, models are wrong
- ❑ **The way you represent geometry matters**
- ❑ Example of  $SO(3)$ 
  - ❑  $r$  is a map from  $E(3)$  to  $E(3)$
  - ❑  $R$  is a orthonormal positive matrix
  - ❑  $w$  is a 3D vector
  - ❑  $q$  is a quaternion represented as a 4D vector
  - ❑ Roll-Pitch-Yaw & other Euler angles should not be used





# Pinocchio bases



1.1 Load and  
display

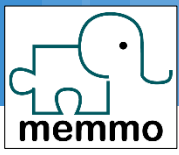


# Load a model

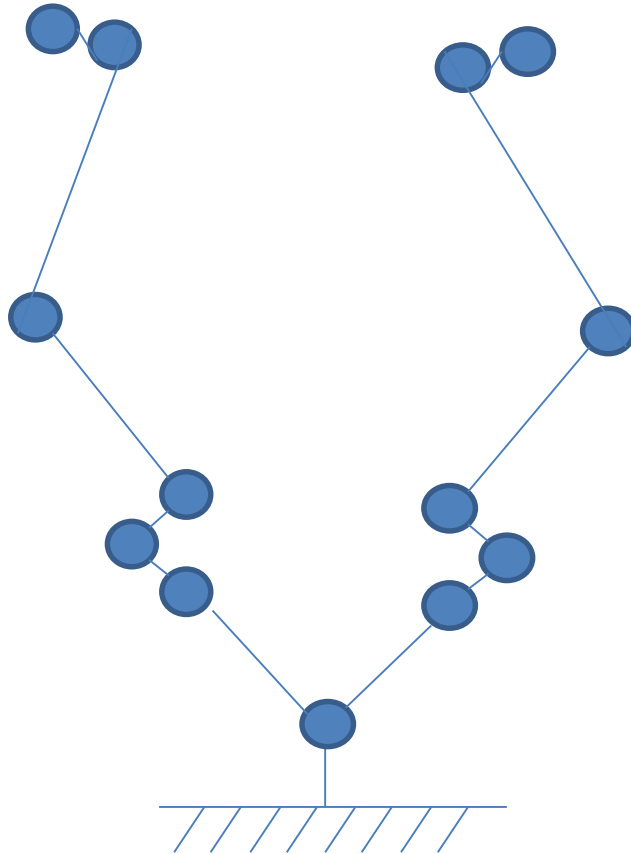
- Pin.buildFromUrdf
- Package example\_robot\_data
  - A small library of our favorite robots
  - Python scripts to load them easily

```
import example_robot_data as robex  
robot = robex.loadTalosArm()
```





# Kinematic tree



Wrist 2

Wrist 1

Elbow

Shoulder 3

Shoulder 2

Shoulder 1

Torso

Universe (joint #0)

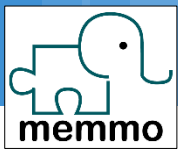
Name  
Type  
Parent  
Placement

Mass  
CoM

Geometries  
Op frames





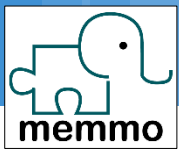


# Kinematic tree

- ❑ Inside robot model:
  - ❑ joints: joint types and indices
  - ❑ names: joint names
  - ❑ jointPlacements: constant placement wrt parent
  - ❑ parents: hierarchy of joints representing the tree
- ❑ No bodies
  - ❑ masses and geoms are attached as tree decorations
- ❑ First joint represent the universe
  - ❑ If  $nq == 7$  then  $\text{len}(\text{rmodel.joints}) == 8$



- ❑ External display servers
  - ❑ Python can create a client to this server
  - ❑ Gepetto viewer
  - ❑ MeshCat
  - ❑ Beta version of a Panda server
  
- ❑ The viewers does not know the kinematic tree
  - ❑ Pinocchio must place the bodies
  - ❑ `pin.visualize` is doing that for you (not in C++)



# Model, data and algorithms

- `pinocchio.Model` should be constant
  - Kinematic tree, joint model, masses, placements ...
  - Plain names used here
- `pinocchio.Data` is modified by the algorithms
  - $oM_i, v, a$
  - $J, J_{com}$
  - $M,$
  - $\tau, nle$
- **1 Model, several Data**

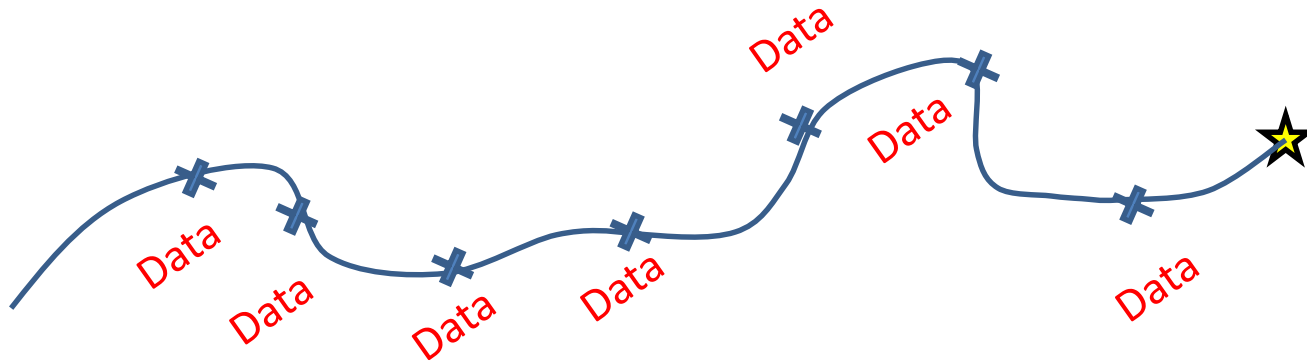




1.2 Pinocchio's  
philosophy

(model, data  
and algos)



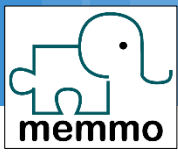


$$\min_{X,U} l_T(x_T) + \sum_{t=0}^{T-1} l(x_t, u_t)$$

← 1 model

$$\text{s.t. } x_{t+1} = f(x_t, u_t)$$





## □ Algorithms:

- With model and data in input
- Store final (and some intermediary) results in data
- Often return the main results

```
pin.randomConfiguration(rmodel)
```

```
pin.forwardKinematics(rmodel, rdata, q)  
rdata.oMi[jointIndex]
```

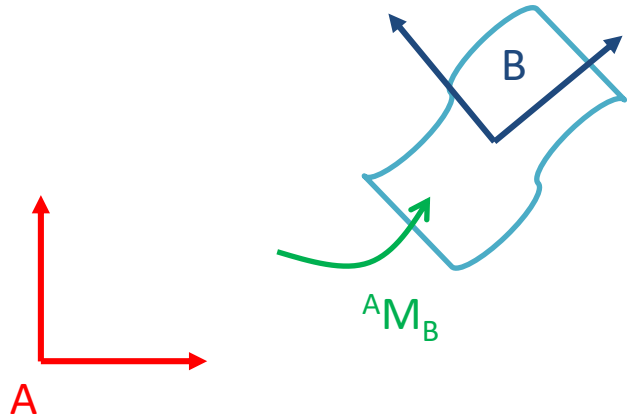




# Forward kinematics

- ❑ `pin.forwardKinematics (rmodel, rdata, q, vq, aq)`
- ❑ `q` -> propagates placements (= forward geometry)
- ❑ `vq` -> also propagates velocity (= *differential* kinematics)
- ❑ `aq` -> also propagates accelerations (= 2<sup>nd</sup> order FK)
  
- ❑ Compute all the joint placements in `data.oMi`
- ❑ `M = data.oMi[jointIndex]` : placement of `<jointIndex>`
- ❑ `R = M.rotation`
- ❑ `p = M.translation`





$${}^A M_B = \begin{bmatrix} {}^A R_B & {}^A \overrightarrow{AB} \\ 0 & 1 \end{bmatrix}$$

$${}^A p = {}^A M_B {}^B p$$

$${}^A M_B {}^B M_C = {}^A M_C$$

`aMb.translation` # 3d array

`aMb.rotation` # 3x3 array





1.3 Cost 3d

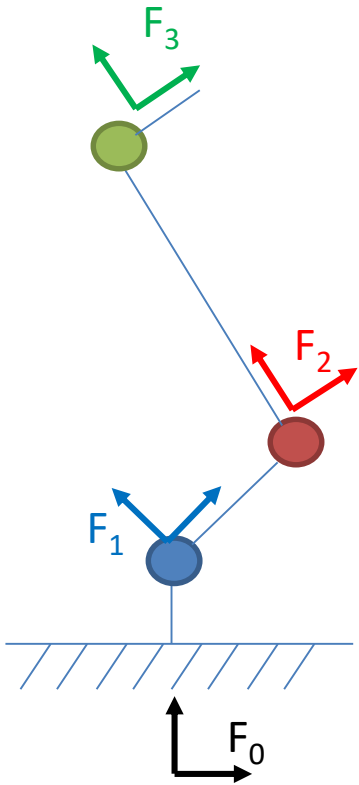
Joint frames

VS

operational  
frames



- ❑ One joint = one joint frame
  - ❑ Attached to the joint output
  - ❑  $F_0$  is the “universe” world frame
- ❑ Operational frames attached to joint frames
  - ❑ Name
  - ❑ Placement
  - ❑ parent





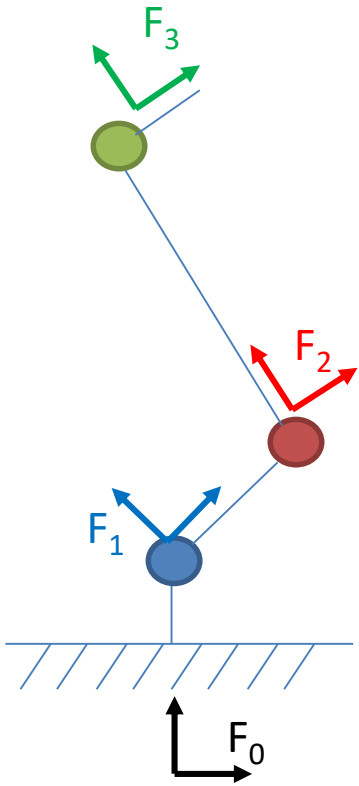
- ❑ Joint frames
  - ❑ Skeleton of the kinematic chain
  - ❑ Computed by forward kinematics in `rdata.oMi`
- ❑ “Operational” frames
  - ❑ Added as decoration to the tree
  - ❑ Placed with respect to a joint parent
  - ❑ Stored in `rmodel.frames`
  - ❑ Computed by `updateFramePlacements` in `rdata.oMf`



```
for f in rmodel.frames:  
    print(f.name, f.parent)
```

```
pin.frameForwardKinematics(rmodel,  
                           rdata, q)
```

```
frameIndex = \  
rmodel.getFrameId('myname')  
rdata.oMf[frameIndex]
```





# Cost model

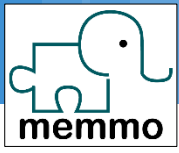
- For this tutorial only (ad-hoc code)
  - ... but similar to the organization in crocodyl

```
class Cost:
    def __init__(self, rmodel, rdata, viz=None):
        self.rmodel = rmodel
        self.rdata = rdata
        self.viz = viz

    def calc(self, q):
        ### Add the code to recompute your cost here
        cost = 0
        return cost

    def callback(self, q):
        if viz is None: return
        # Display something in viz ...
```

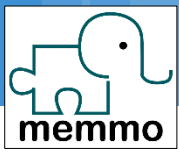




# SciPy optimizer

- ❑ Make the optimization problem a class:
  - ❑ Problem parameters in the `__init__`
  - ❑ Cost method taking `x` as input
  - ❑ Gradient and callback method if need be





```
class OptimProblem:
```

```
    def __init__(self,rmodel):
```

```
        # Put your parameters here
```

```
        self.rmodel = rmodel
```

```
        self.rdata = self.rmodel.createData()
```

```
    def cost(self,x): return sum( x**2 )
```

```
    def callback(self,x): print(self.cost(x))
```

```
pbm = OptimProblem(robot.model)
```

```
fmin_slsqp(x0=x0,func=pbm.cost,callback=pbm.callback)
```



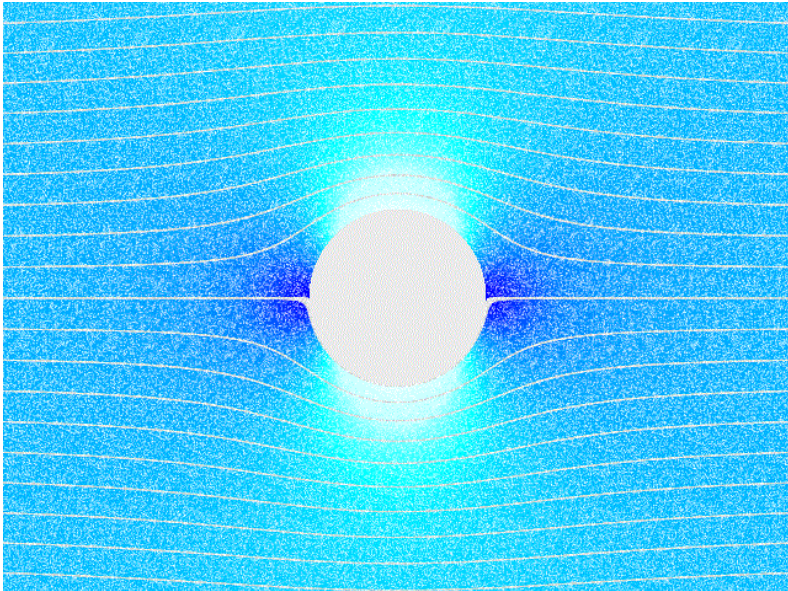


1.4 Cost  $\phi_d$

$SE(3)$  and log



- Between 2 frames:  
Each point  
moves to another point



$$m: p \in E^3 \rightarrow m(p) \in E^3$$

[https://upload.wikimedia.org/wikipedia/commons/b/b8/Inviscid\\_flow\\_around\\_a\\_cylinder.gif](https://upload.wikimedia.org/wikipedia/commons/b/b8/Inviscid_flow_around_a_cylinder.gif)

□ Between 2 frames:

Each point

moves to another point

Distances are kept

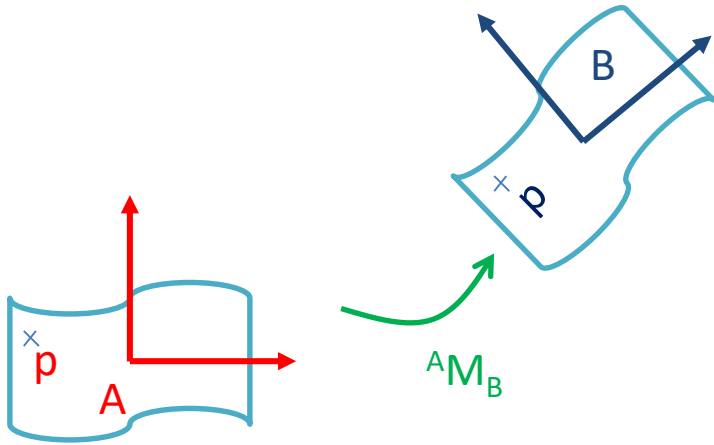
Angles are kept



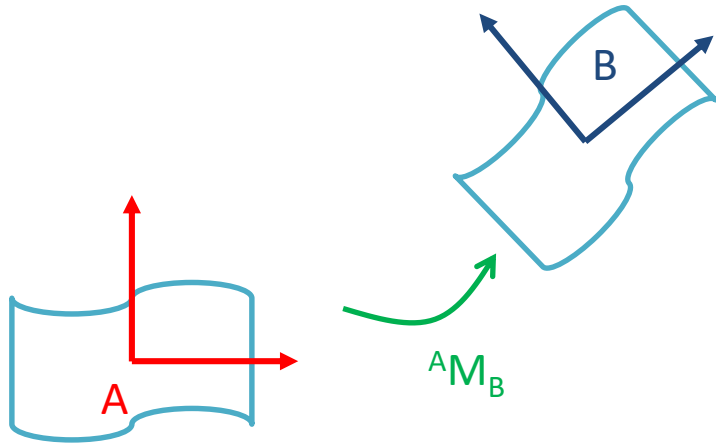
<https://gfycat.com/fr/sleepycleanarcherfish>

$$m: p \in E^3 \rightarrow m(p) \in E^3$$

- ${}^A M_B = ({}^A R_B, {}^A A_B)$  represents the motion of all the points of the body



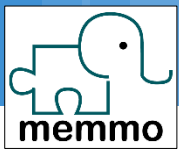
$${}^A p = {}^A M_B {}^B p$$



- Rigid velocities  
= linear + angular velocity
- What is the velocity to transform  $\mathcal{F}_A$  into  $\mathcal{F}_B$  in 1 second ?

“SE(3) Logarithm”  $\log({}^A M_B)$

```
M=pin.SE3.Random()  
pin.log(M).vector
```



# Position versus placement

- Difference of positions

- residuals =  $p - p^*$

- Difference of rotations

- residuals =  $\log_3( R^T R^* )$

$pin.log3$

- Difference of placements

- residuals =  $\log_6( M^{-1} M^* )$

$pin.log6$

$pin.log$   
(auto-switch  
based on type)





jupyter

1.5 redundancy

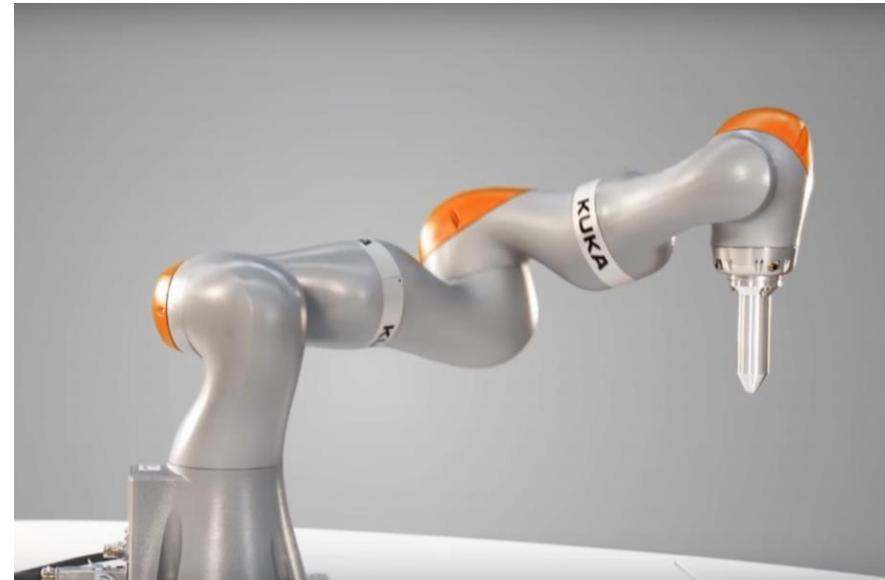
Posture cost



<https://youtu.be/sZYBC8Lrmdo>

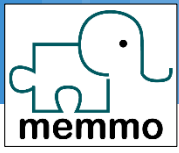
MEMMO: Memory of Motion – [www.memmo-project.eu](http://www.memmo-project.eu)





- ❑ Same 6D cost
- ❑ Hessian is ill-defined



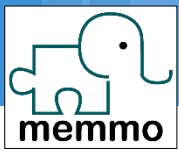


# Sum of costs

$$c(\mathbf{q}) = w_1 c_1(\mathbf{q}) + w_2 c_2(\mathbf{q})$$

$$\nabla c(\mathbf{q}) = w_1 \nabla c_1(\mathbf{q}) + w_2 \nabla c_2(\mathbf{q})$$





# SciPy optimizer

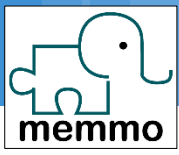
```
from scipy.optimize import fmin_bfgs  
fmin_bfgs?
```

```
fmin_bfgs(x0 = np.zeros(7),  
          func= costFunction,  
          fprime = gradFunction,  
          callback=callbackFunction)
```





## 1.6 Dynamics

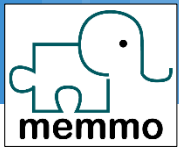


# Whole body dynamics

$$\mathbf{M}(\mathbf{q}) \mathbf{a}_{\mathbf{q}} + \mathbf{b}(\mathbf{q}, \mathbf{v}_{\mathbf{q}}) = \boldsymbol{\tau}_{\mathbf{q}}$$

Explain  $\mathbf{v}_{\mathbf{q}}$ ,  $\mathbf{a}_{\mathbf{q}}$ ,  $\boldsymbol{\tau}_{\mathbf{q}}$ ,  $\mathbf{M}$ ,  $\mathbf{b}$ ,



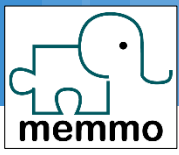


# Whole body dynamics

$$\mathbf{M}(\mathbf{q}) \mathbf{a}_{\mathbf{q}} + \mathbf{b}(\mathbf{q}, \mathbf{v}_{\mathbf{q}}) = \boldsymbol{\tau}_{\mathbf{q}}$$

Explain Lagrange  $\frac{d}{dv} \dot{L} - \frac{d}{dq} L$





# Whole body dynamics

$$\mathbf{M}(\mathbf{q}) \mathbf{a}_{\mathbf{q}} + \mathbf{b}(\mathbf{q}, \mathbf{v}_{\mathbf{q}}) = \boldsymbol{\tau}_{\mathbf{q}}$$

Explain b and g



## □ Gravity

$$g(q) = b(q, v_q=0)$$

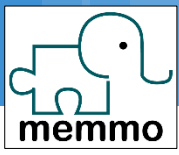
```
pin.computeGeneralizedGravity \  
(rmodel,  
 rdata, q)
```



1.6 Weighted  
gravity







- Inverse dynamics

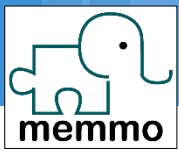
$$\tau_q = \text{invdyn}(q, v_q, a_q)$$

- Direct / forward dynamics

$$a_q = \text{dirdyn}(q, v_q, \tau_q)$$

Explain control / simu ... explicit invdyn / dirdyn equations





## □ Inverse dynamics

```
tauq = pin.rnea(rmodel, rdata,  
               q, vq, aq)
```

## □ Direct / forward dynamics

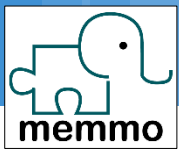
```
aq = pin.aba(rmodel, rdata,  
            q, vq, tauq)
```

## □ Generalized inertia “mass” matrix

```
M = pin.crab(rmodel, rdata, q)
```

Give timings





# Weighted gravity

$$c(q) = g(q)^T M(q)^{-1} g(q)$$

Compute  $c$  from  $r_{nea}$  and  $aba$

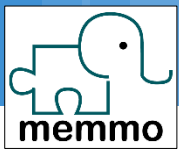




1.8 floating  
basis

Humanoids and  
quadrupeds

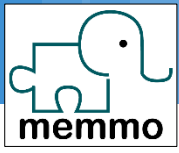




# Free flyer joint

- Revolute joint
  - $q$  of dimension one,  $v_q = \dot{q}$
- Free flyer





# Integrate and differentiate

$$q_{\text{next}} = \text{pin.integrate}(q, v_q) \in Q$$

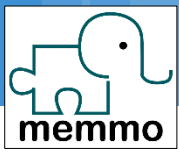
$$q_{\text{next}} = q \oplus v_q$$

$$\Delta q = v_q = \text{pin.difference}(q_1, q_2) \in T_{q_1}Q$$

$$\Delta q = q_2 (-) q_1$$

$$q = \text{pin.normalize}(\text{rand}(nq)) \in Q$$

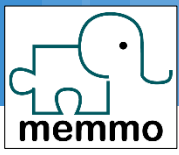




# Optimization with Q / TQ

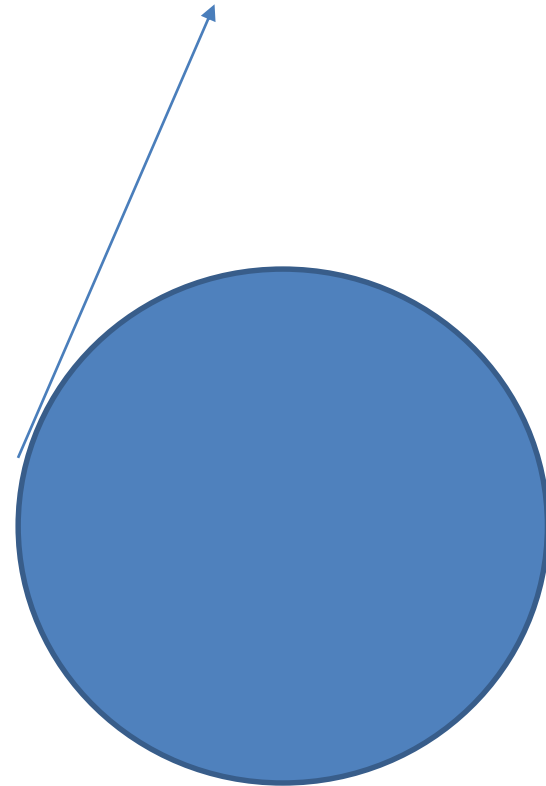
- $q = (x, y, z, \underline{q}, \dots)$  with  $\underline{q}$  unitary
- What is the result with a solver ?



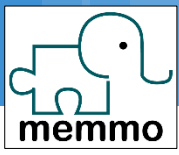


# Solution 1: normalized

```
def constraint_q(self, x):  
    return norm(x[3:7])-1
```







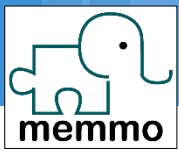
# Solution 2: normalize

```
def cost(q):
```

```
    q = pin.normalize(rmodel, q)
```

```
    ... # compute the cost
```





# Solution 3: reparametrize

- We represent  $q$ 
  - as the displacement  $v_q$
  - from a reference configuration  $q_0$

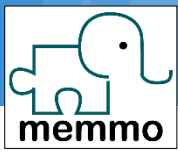
$$q = q_0 \oplus v_q$$





Fin ?

The End ?



# Exercise

- ❑ With humanoid Talos or quadruped Solo
  
- ❑ Choose contact location
  - ❑ 3d contacts for the quadruped or the humanoid hand
  - ❑ 6d contacts for the humanoid feet
  
- ❑ From a random configuration ...
  - ❑ Optimize the 3d/6d costs + posture cost
  - ❑ Project the feet in contact

