Task-Space Inverse Dynamics: Implementation

Optimization-based Robot Control

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Introduction

Key Concepts

Task

- Motion
- Force
- Actuation

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HQP Solver

- solves a HQP

Other Concepts

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Trajectory

- maps time to vector values
- pos, vel, acc
- position and velocity can have different sizes (Lie groups)

Details

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- Inequalities, represented by matrix A and vectors lb and ub:

$$lb \leq Ax \leq ub$$

- Bounds, represented by vectors lb and ub:

$$lb \leq x \leq ub$$

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Interface of `TaskBase`:

```
TaskBase(string name, Model model);
Constraint compute(double t, Vector q, Vector v, Data data);
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- `TaskContactForce`: linear function of contact forces
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- bounds, e.g., `TaskJointBounds`
- inequality constraints, e.g., friction cones

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Matrix computeForceGeneratorMatrix();
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Force task:

- represents inequality constraints acting on contact forces
- e.g., friction cone constraints
- $Af \leq a$

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- e.g., keep them close to friction cone center

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Force-Generator matrix T:

- maps force variables to motion constraint representation
- Dynamic: $M\dot{v} + h = S^\top \tau + J^\top T f$
- Motion constraint: $J\dot{v} = -\dot{J}v$
- Friction cones: $Af \leq a$

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- unilateral plane contact $\rightarrow$ 6d motion constraint
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SOLUTION

- use 6d representation for motion constraint $J\dot{v} = -\dot{J}v \in \mathbb{R}^6$

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SOLUTION

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- but 12d representation for force variable $f \in \mathbb{R}^{12}$
- force-generator matrix $T \in \mathbb{R}^{6\times 12}$ defines mapping between two representations: $\tau_{contact} = J^\top T f$

InverseDynamicsFormulationBase

Central class of the whole library

Methods to add tasks:

```
addMotionTask(MotionTask task, double weight, int priority);
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HqpData defined as:

```
#typedef vector<pair<double, ConstraintBase>> ConstraintLevel
#typedef vector<ConstraintLevel> HqpData
```

Examples

Exercise 1

Open Terminal and execute:

```
cd $TSID_HOME
git pull
cd exercizes/notebooks
jupyter notebook
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- Add contact on hand

Exercise 2: Balancing

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